Interfacial separation between elastic solids with randomly rough surfaces: comparison of experiment with theory

Interfacial separation between elastic solids with randomly rough surfaces: comparison of experiment with theory

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Abstract
We study the average separation between an elastic solid and a hard solid, with a nominally flat but randomly rough surface, as a function of the squeezing pressure. We present experimental results for a silicon rubber (PDMS) block with a flat surface squeezed against an asphalt road surface. The theory shows that an effective repulsive pressure acts between the surfaces of the form $p \sim \exp(-u/u_0)$, where u is the average separation between the surfaces and u_0 a constant of the order of the root-mean-square roughness, in good agreement with the experimental results.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Contact mechanics between solid surfaces is the basis for understanding many tribology processes [1–7] such as friction, adhesion, wear, and sealing. The two most important properties in contact mechanics are the area of real contact and the interfacial separation between the solid surfaces. For non-adhesive contact and small squeezing pressure, the (projected) contact area depends linearly on the squeezing pressure [8–11].

When two elastic solids with rough surfaces are squeezed together, the solids will in general not make contact everywhere in the apparent contact area, but only at a distribution of asperity contact spots. The separation $u(\mathbf{x})$ between the surfaces will vary in a nearly random way with the lateral coordinates $\mathbf{x} = (x, y)$ in the apparent contact area. When the applied squeezing pressure increases, the average surface separation $u = \langle u(\mathbf{x}) \rangle$ will decrease, but in most situations it is not possible to squeeze the solids into perfect contact corresponding to $u = 0$. We have recently developed a theory which predicts that, for randomly rough surfaces at low squeezing pressures, $p \sim \exp(-u/u_0)$, where the reference length u_0 depends on the nature of the surface roughness but is independent of p [1, 12]. Here we will present experimental results to test the theoretical predictions[1]. We study the squeezing of a rubber block against an asphalt road

surface. This topic is also important in the context of the air-pumping contribution to tire noise [14]. Thus the compression and outward flow of the air between a tread block and the road surface cavities during driving contribute to tire noise, similarly to how sound is generated during applause. A similar effect (but now involving decompression and inward flow of air) occurs when a tread block leaves the tire–road contact area.

2. Theory

We consider the frictionless contact between an elastic solid (elastic modulus E and Poisson ratio ν) with a flat surface and a rigid, randomly rough surface with the surface height profile $z = h(\mathbf{x})$. The separation between the average surface plane of the block and the average surface plane of the substrate (see figure 1) is denoted by u with $u \geqslant 0$. When the applied squeezing force p increases, the separation between the surfaces at the interface will decrease, and we can consider $p = p(u)$ as a function of u. The elastic energy $U_{\text{el}}(u)$ stored in the substrate asperity–elastic block contact regions must be equal to the work done by the external pressure p in displacing the lower surface of the block towards the substrate. Thus,

$$p(u) = -\frac{1}{A_0} \frac{\mathrm{d}U_{\text{el}}}{\mathrm{d}u}, \qquad (1)$$

where A_0 is the nominal contact area. For elastic solids equation (1) is exact [12, 15]. The equation holds also for

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[1] Experiments involving the squeezing of rubber blocks against rough surfaces have been performed by Gäbel and Kröger [13] but without comparing the experimental results to theory.
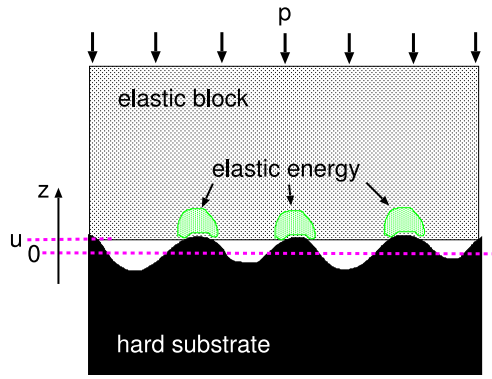
Figure 1. An elastic block squeezed against a rigid rough substrate. The separation between the average plane of the substrate and the average plane of the lower surface of the block is denoted by u. Elastic energy is stored in the block in the vicinity of the asperity contact regions.

viscoelastic solids if the compression occurs so slowly that negligible energy dissipation (caused by the internal friction of the solids) occurs during the compression. In our experiments we use silicon rubber which behaves as a perfect elastic solid under our experimental conditions.

Theory shows that for low squeezing pressure, the area of real contact A varies linearly with the squeezing force pA_0, and that the interfacial stress distribution and the size distribution of contact spots are independent of the squeezing pressure [16]. That is, with increasing p existing contact areas grow and new contact areas form in such a way that in the thermodynamic limit (infinite-sized system) the quantities referred to above remain unchanged. It follows immediately that for small load *the elastic energy stored in the asperity contact region will increase linearly with the load*, i.e. $U_{el}(u) = u_0 A_0 p(u)$, where u_0 is a characteristic length which depends on the surface roughness (see below) but is independent of the squeezing pressure p. Thus, for small pressures (1) takes the form

$$p(u) = -u_0 \frac{dp}{du}$$

or[2]

$$p(u) \sim e^{-u/u_0}. \qquad (2)$$

To quantitatively derive the relation $p(u)$ we need an analytical expression for the asperity induced elastic energy. Within the contact mechanics approach of Persson we have [16, 18, 19]

$$U_{el} \approx A_0 E^* \frac{\pi}{2} \int_{q_0}^{q_1} dq \, q^2 P(q, p) C(q), \qquad (3)$$

where $E^* = E/(1 - v^2)$ and where $P(q, p) = A(\zeta)/A_0$ is the relative contact area when the interface is studied at the magnification $\zeta = q/q_0$, which depends on the applied
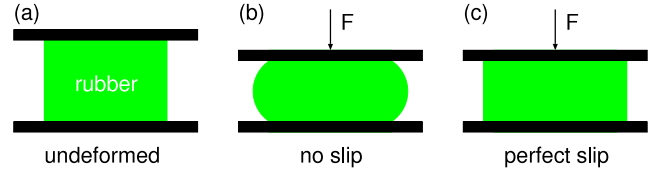
Figure 2. A rubber block between two flat and rigid solid plates. (a) Undeformed state. (b) Squeezed block assuming no slip (i.e. high enough static friction) at the rubber–plate interfaces. (c) Squeezed block assuming perfect slip (i.e. no friction) at the rubber–plate interfaces.

pressure p. The surface roughness power spectrum [16]

$$C(q) = \frac{1}{(2\pi)^2} \int d^2x \, \langle h(\mathbf{x})h(\mathbf{0})\rangle e^{-i\mathbf{q}\cdot\mathbf{x}},$$

where $\langle \cdots \rangle$ stands for ensemble average. Note that for complete contact $P = 1$ and in this limit (3) is exact. For self-affine fractal surfaces the prediction of the contact mechanics theory of Persson has been compared to numerical simulations [19, 20]. The numerical studies indicate that as the fractal dimension of the surface approaches 2 the Persson theory may become exact, while a small difference between theory and simulations is observed for larger fractal dimensions [21]. Below we will compare the theoretical predictions with experimental data for an asphalt road surface which is fractal-like with the fractal dimension $D_f \approx 2$. We find nearly perfect agreement between theory and experiment (see below), supporting the picture gained before based on numerical simulations.

Substituting (3) in (1) gives for small squeezing pressures [12]:

$$p = \beta E^* e^{-u/u_0}. \qquad (4)$$

For self-affine fractal surfaces, the length u_0 and the parameter β depend on the Hurst exponent H and on q_0 and q_1. Most surfaces which are self-affine fractal have the Hurst exponent $H > 0.5$ (or the fractal dimension $D_f < 2.5$). For such surfaces u_0 and β are nearly independent of the highest surface roughness wavevector, q_1, included in the analysis. For the substrate surface studied below we obtain from the measured surface roughness power spectrum (see figure 7) $u_0 = 0.30$ mm and $\beta = 0.59$. Note that u_0 is of the order of the root-mean-square roughness amplitude ($h_{rms} \approx 0.29$ mm in the present case, see below).

Consider a rubber block (elastic modulus E) with a flat surface (area A_0) and thickness d. We will study both dry and lubricated interfaces (see figure 2) resulting in no slip and perfect slip at the two rubber-confining wall interfaces. If the block is squeezed against a rigid, randomly rough counter surface, the upper surface of the rubber block will move downwards by the distance s (see figure 3), which is the sum of a uniform compression of the rubber block, $d\sigma/E$, and a movement (or penetration) w of the average position of the lower surface of the rubber block into the valleys or cavities of the countersurface:

$$s = w + d\sigma/E. \qquad (5)$$

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[2] We note that the result (2) differs drastically from the prediction of asperity contact mechanics theories such as those of Bush *et al* [17] and the theory of Greenwood and Williamson [17].
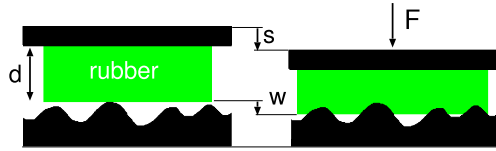
Figure 3. A rubber block in contact with a rigid, randomly rough substrate. Left: no applied load. Right: the rubber block is squeezed against the substrate with force F. The upper and (the average position of) the lower surface of the rubber block moves downwards by the distances s and w, respectively. We assume perfect interfacial slip (no friction).

If u denote the average separation between the block and the substrate (so that $u = 0$ corresponds to perfect contact) then

$$w = h_{max} - u \qquad (6)$$

where we have assumed that the initial position of the lower surface of the block corresponds to the separation where the block just makes contact with the highest substrate asperity (as in figure 3, left), which is located a distance h_{max} above the average substrate surface plane. Using (4) we get

$$\log(\sigma/E) = \log(4\beta/3) - u/u_0 \qquad (7)$$

where $\sigma = F/A_0$ the squeezing pressure. Here we have used $E^*/E = 1/(1 - \nu^2) \approx 4/3$ since for rubber $\nu \approx 1/2$. Combining (5) and (6) gives

$$u = h_{max} - s + d\sigma/E.$$

Substituting this in (7) gives

$$\log\left(\frac{\sigma}{E}\right) = \log\left(\frac{4\beta}{3}\right) - \frac{1}{u_0}\left(h_{max} - s + d\frac{\sigma}{E}\right)$$

or

$$\log\left(\frac{\sigma}{E}\right) = B + \frac{1}{u_0}\left(s - d\frac{\sigma}{E}\right) \qquad (8)$$

where $B = \log(4\beta/3) - h_{max}/u_0$.

For the no-slip boundary condition, equation (5) is replaced by

$$s = w + d\sigma/E'$$

where the effective modulus $E' > E$. Thus, in this case (8) takes the form

$$\log\left(\frac{\sigma}{E'}\right) = B' + \frac{1}{u_0}\left(s - d\frac{\sigma}{E'}\right) \qquad (9)$$

where $B' = \log(4\beta E/3E') - h_{max}/u_0$.

3. Experimental details

To test the theory presented above, we have performed the experiment indicated in figure 3. A rubber block with a flat surface was squeezed against an asphalt road surface. The displacement s of the upper surface of the rubber block was changed in steps of 0.05 mm, and the force F was measured. For the experiment we used a test stand produced by SAUTER

GmbH (Albstadt, Germany), normally used to measure spring constants. Using this test stand, we were able to measure forces up to 500 N, and displacement with a resolution of 0.01 mm.

The rubber block was made from a silicone elastomer (PDMS). The PDMS samples were prepared using a two-component kit (Sylgard 184) purchased from Dow Corning (Midland, MI). This kit consists of a base (vinyl-terminated polydimethylsiloxane) and a curing agent (methylhydrosiloxane–dimethylsiloxane copolymer) with a suitable catalyst. From these two components we prepared a mixture 10:1 (base/cross linker) in weight. The mixture was degassed to remove the trapped air induced by stirring from the mixing process and then poured into cylindrical casts (diameter $D = 3$ cm and height $d = 1$ cm). The bottom of these casts was made from glass to obtain smooth surfaces (negligible roughness). The samples were cured in an oven at 80 °C for over 12 h.

The road surface used in this experiment was provided by Pirelli (Italian tire manufacturer). The topography was measured with contact-less optical methods using a chromatic sensor with two different optics produced by Fries Research and Technology GmbH (Bergisch Gladbach, Germany). To identify the elastic modulus E, the PDMS sample was first squeezed against a smooth substrate in a compression test. We measured the force F over the displacement s for two different cases. First there was no lubrication used and the PDMS sample deformed laterally at the force-free area, as shown in figure 2(b), because no slip occurred at the contact areas. Second we lubricated the contact areas to obtain perfect slip at the interfaces (see figure 2(c)). We used polyfluoroalkylsiloxane (PFAS), a fluorinated silicone oil supplied by ABCR GmbH & Co. KG (Karlsruhe, Germany). Because of its high viscosity ($\eta = 1000$ cSt), the fluid is an excellent lubricant also under extreme pressure applications and should therefore not easily be squeezed out of the contact area. Also it does not react (or interdiffuse) with the PDMS elastomer.

4. Results

Consider first flat surfaces. In figure 4 we show the measured relation between the stress and the strain for lubricated surfaces (so that the shear stress vanish on the boundaries). If the stress is normalized with $E = 2.3$ MPa, a nearly straight line with slope 1 will result, so that the relation $\sigma = Es/d$ holds. The elastic modulus $E = 2.3$ MPa is consistent with the elastic modulus reported in the literature for similar silicon rubbers[3]. We note that when repeating this experiment (figure 4), as well as the other similar experiments described below, the new results never differed by more than ∼2.5% from the original measurements.

We have also performed experiments for dry surfaces. In this case no (or negligible) slip occurred at the interface with the confining walls, and visual inspection of the system

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[3] See, e.g. Bongaerts *et al* [22], where they report a Young's modulus $E = 2.4$ MPa for PDMS prepared in the same way as ours, using Sylgard 184 with a base/curing agent mass ratio of 10:1. Similarly, Scheibert *et al* [22] obtained the Young's modulus 2.2 ± 0.1 MPa.
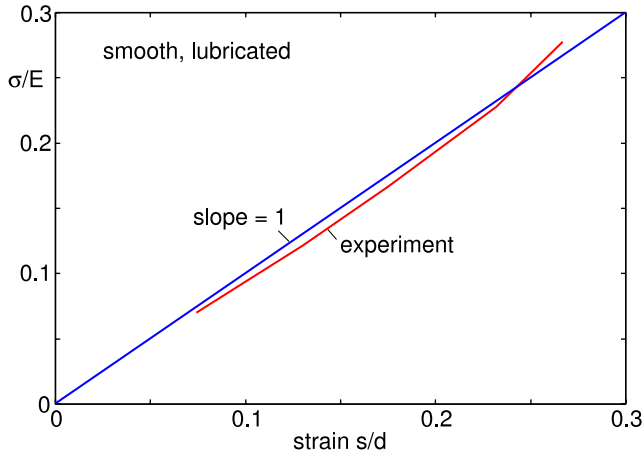
Figure 4. The stress σ (in units of the elastic modulus E) as a function of the strain s/d, where s is the displacement of the upper surface and d the thickness of the block. In the calculation we used $E = 2.3$ MPa for a PDMS rubber block confined between two smooth lubricated (wet) surfaces.
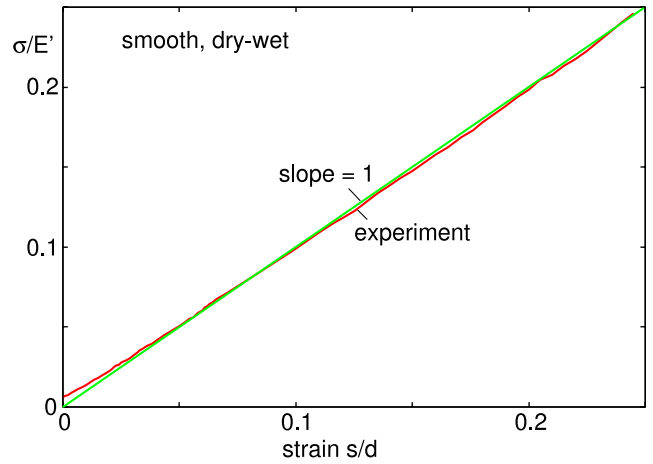

Figure 6. The stress σ (in units of the elastic modulus E') as a function of the strain s/d, where s is the displacement of the upper surface and d the thickness of the block. In the calculation we used the effective modulus $E' = 2.9$ MPa. For a PDMS rubber block confined between one lubricated (wet) surface and one dry surface.
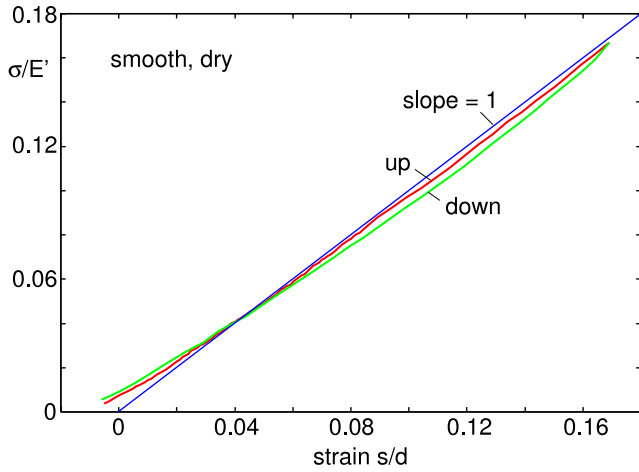

Figure 5. The stress σ (in units of the elastic modulus E') as a function of the strain s/d, where s is the displacement of the upper surface and d the thickness of the block. In the calculation we used the effective modulus $E' = 4.2$ MPa for a PDMS rubber block confined between smooth dry surfaces. The two experimental curves correspond to increasing and decreasing strain.

showed that the rubber bulged laterally at the force-free area (see figure 2(b)). We still expect a linear (or near linear) relation between stress and strain but the effective elastic modulus E' is larger than for lubricated interfaces. Thus, the effective elastic modulus deduced from the experimental data (see figure 5) $E' \approx 4.2$ MPa is about 80% larger than for the lubricated interface. To check the measuring system for hysteresis effects, some of the experiments were performed bidirectionally. The results are shown in figure 5 where the strain was increased and after that slowly decreased again. Negligible hysteresis occurred, as expected because of the low glass transition temperature of PDMS.

The increase in the effective elastic modulus in compression, from 2.3 to 4.2 MPa, when going from slip to no-slip boundary conditions, is consistent with the prediction

of the Lindley equation [23], which in the present case takes the form

$$E' \approx E \left(1 + 1.4 S^2\right)$$

For a cylinder, the *shape factor* $S = R/2d$. In the present case $E = 2.3$ MPa and $S = 0.75$ giving $E' = 4.1$ MPa which agrees very well with the measured value (4.2 MPa).

We have also studied the case where one surface is lubricated and the other dry. In this case the rubber will displace laterally in an asymmetric way (as in figure 10(b)) and the measured effective elastic modulus $E' = 2.9$ MPa (see figure 6), is slightly smaller than the average of the effective E-modulus obtained assuming no slip and complete slip on both surfaces: $(2.3 + 4.2)/2$ MPa ≈ 3.3 MPa.

We will now present experimental results for a rubber block squeezed against an asphalt road surface. The surface roughness power spectrum of the road surface is shown in figure 7. The surface has the root-mean-square roughness $h_{\mathrm{rms}} \approx 0.29$ mm, and for the wavevector $q > q_0 \approx 2500$ m^{-1} it is (on a log–log scale) well approximated by a straight line with the slope corresponding to a self-affine fractal surface with the fractal dimension $D_{\mathrm{f}} = 2$. For $q < q_0$, $C(q)$ is approximately constant; we refer to q_0 as the roll-off wavevector.

In figure 8 we show the natural logarithm of the squeezing pressure (divided by the effective elastic modulus) as a function of $s - d\sigma/E'$, where s is the displacement of the upper surface of the rubber block relative to the substrate, and where d is the thickness of the rubber block. In the calculation we used the effective elastic modulus $E' = 4.8$ MPa and $B' = -6.85$. The value of B' has been calculated using (9) (using the measured h_{max}) so that the only fitting parameter is the effective elastic modulus E', which, however, agrees rather well with the measurements for flat surfaces ($E' = 4.2$ MPa).

In figure 9 we show the same as in figure 8 but now for lubricated surfaces. In the calculation we used the effective elastic modulus $E' = 3.4$ MPa and $B' = -6.50$. Note that
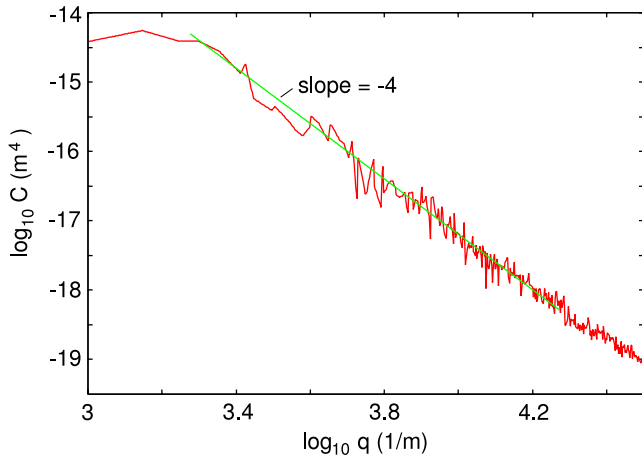
Figure 7. The surface roughness power spectrum C, as a function of the wavevector q (log–log scale), for an asphalt road surface. The straight green line has the slope -4, corresponding to the Hurst exponent $H = 1$ (fractal dimension $D_f = 2$).
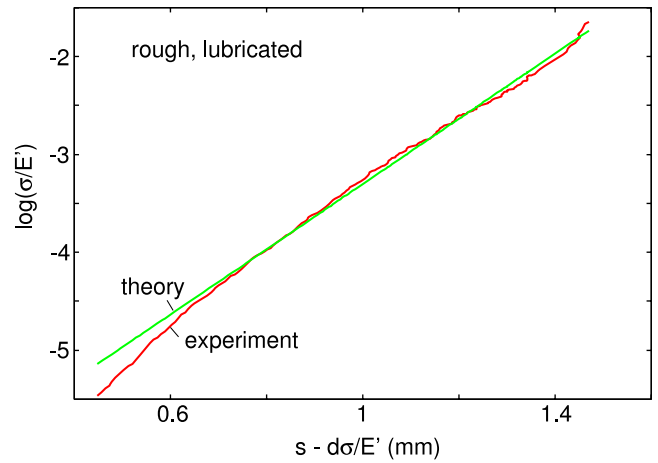

Figure 8. The natural logarithm of the squeezing pressure (divided by the effective elastic modulus) as a function of $s - d\sigma/E'$, where s is the displacement of the upper surface of the rubber block relative to the substrate, and where d is the thickness of the rubber block. In the calculation we used the effective elastic modulus $E' = 4.8$ MPa and $B' = -6.85$. The two experimental curves were obtained using two different silicon rubber blocks, produced in the same way. The results are for dry contact.

this value for B' is slightly smaller than for dry contacts. The difference $\Delta B' = -6.50 - (-6.85) = 0.35$ just reflects the difference in the effective E-modulus since according to (9) $\Delta B' = \log[E'(\text{dry})/E'(\text{lubricated})] = \log(4.8/3.4) \approx 0.35$. The E' value is larger than the E-modulus measured for flat lubricated surfaces ($E = 2.3$ MPa), but this can be understood as follows.

Visual inspection of the contact between the rubber cylinder and the two confining walls shows that, as expected from above, the rubber block slips against the top (flat) steel surface, while no slip (or only very limited slip) occurs against the rough substrate surface, see figure 10(b). This is consistent with the fact that the observed elastic modulus is larger than $E = 2.3$ MPa, as obtained above when complete slip occurs

Figure 9. The natural logarithm of the squeezing pressure (divided by the effective elastic modulus) as a function of $s - d\sigma/E'$, where s is the displacement of the upper surface of the rubber block relative to the substrate, and where d is the thickness of the rubber block. The results are for lubricated (wet) contact. In the calculation we used the effective elastic modulus $E' = 3.4$ MPa and $B' = -6.5$.
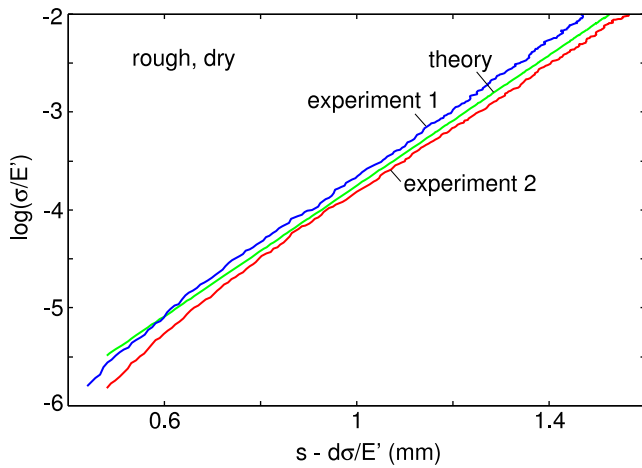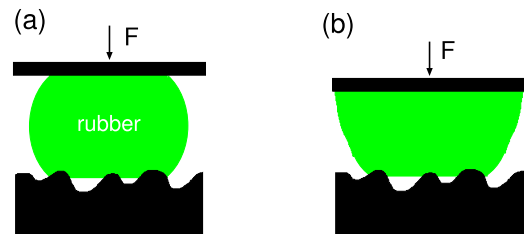

Figure 10. A rubber block squeezed between a rigid solid plate and a rigid randomly rough substrate: (a) dry surfaces and (b) lubricated surfaces.

at both (lubricated) surfaces. In fact, the observed effective E-modulus (3.4 MPa) is quite close to the value 2.9 MPa measured for smooth surfaces when slip occurs at one surface and no slip at the other surface. The fact that no (or very small) slip occurs at the interface between the rubber and the rough substrate surface may be due to at least two facts.

(1) The pressures in the asperity contact regions are much higher than the average pressure, and the asperity contact regions much smaller than the nominal contact area, resulting in much faster squeeze out of the lubricant oil from the asperity contact regions, as compared to the case of flat surfaces, and consequently to higher friction in the contact regions.

(2) The substrate surface roughness on different length scales contributes to the friction during slip because of the viscoelastic deformations of the rubber on different length scales. However, since for silicon rubber viscoelastic dissipation only occurs at very high frequencies, it is likely that this effect is small in the present case.

The measured E'-values for rough surfaces (4.8 and 3.4 MPa) are roughly 14% larger than for smooth surfaces (4.2 and 2.9 MPa), as obtained assuming no slip on the confining

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surfaces in one case, and slip on only one of the confining surfaces in the other case. The origin of this (small) difference in effective elastic modulus is not known to us.

Finally, we note that for $s - d\sigma/E' < 0.6$ mm the experimental curve in figure 9 drops off faster with decreasing interfacial separation than predicted by the theory. (The same effect can also be seen in figure 8 and has also been observed in molecular dynamics calculations [15].) This is a finite size effect: the theory is for an infinite system which has (arbitrary many) arbitrary high asperities, and contact between the two solids will occur for arbitrary large surface separation, and the relation $p \sim \exp(-u/u_0)$ holds for arbitrary large u. On the other hand a finite system has asperities with height below some finite length h_{max}, and for $u > h_{max}$ no contact occurs between the solids and $p = 0$.

5. Summary and conclusion

We have presented a combined experimental–theoretical study of the contact between a rigid solid with a randomly rough surface and an elastic block with a flat surface. The interfacial separation as a function of the squeezing pressure has been derived theoretically and has been compared to the experimental results. We find nearly perfect agreement between theory and experimental data for an asphalt road surface. We conclude that for non-adhesive interaction and small applied pressure, $p \sim \exp(-u/u_0)$, where p is the squeezing pressure and u the average interfacial separation, and u_0 a constant of the order of the root-mean-square roughness of the combined surface profile. In addition, the experimental results indicate that for surfaces with fractal-like roughness profiles the Persson contact mechanics theory may be exact for the fractal dimension $D_f = 2$. We plan to extend the study above to surfaces with other fractal dimensions to test the theory in more general cases. The presented results may be of great importance for, for example, heat transfer, lubrication, sealing, optical interference, and tire noise related to air-pumping.

Acknowledgments

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